

Find the rectangular equation of the ellipse with foci $(-7, 5)$ and $(-7, -1)$ and minor axis of length 12.

SCORE: ____ / 15 PTS

$$\text{CENTER } (-7, \frac{5+(-1)}{2}) = (-7, 2)$$



$$\frac{(x+7)^2}{36} + \frac{(y-2)^2}{45} = 1$$

$$3^2 + 6^2 = 45$$

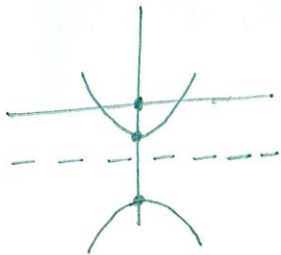
A hyperbola has a focus at the pole and vertices with rectangular co-ordinates $(0, -2)$ and $(0, -7)$.

SCORE: ____ / 20 PTS

- [a] Find polar co-ordinates for the vertices, using positive values of r and θ .

$$(2, \frac{3\pi}{2}) \quad (7, \frac{3\pi}{2})$$

- [b] Find the polar equation of the hyperbola. Do NOT find the rectangular equation.



$$r = \frac{ep}{1 - e \sin \theta}$$

$$\text{USE } (-7, \frac{\pi}{2}) \text{ AND } (2, \frac{3\pi}{2})$$

$$-7 = \frac{ep}{1 - e}$$

$$2 = \frac{ep}{1 + e}$$

$$ep = -7 + 7e$$

$$ep = 2 + 2e$$

$$-7 + 7e = 2 + 2e$$

$$e = \frac{9}{5}$$

$$ep = 2 + 2\left(\frac{9}{5}\right) = \frac{28}{5}$$

$$r = \frac{\frac{28}{5}}{1 - \frac{9}{5} \sin \theta}$$

$$r = \frac{28}{5 - 9 \sin \theta}$$

Consider the polar equation $r = 3 + 2\sin \theta$.

$$1 < \left| \frac{3}{2} \right| < 2$$

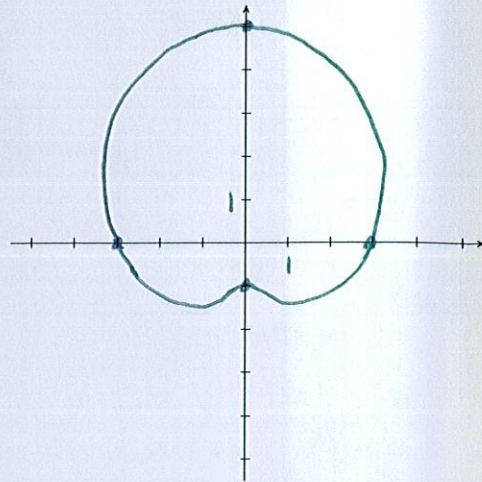
SCORE: ____ / 20 PTS

- [a] What is the shape of the graph of the equation ?

LIMACON WITH DIMPLE
↳

- [b] Sketch the graph using the shortcut process shown in lecture / PowerPoint.
Label the scale on your axes clearly.

θ	r
0	3
$\frac{\pi}{2}$	5
π	3
$\frac{3\pi}{2}$	1



Convert the polar equation $r = 1 + \tan \theta$ to rectangular and simplify.

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$$\sqrt{x^2 + y^2} = 1 + \frac{y}{x}$$

$$x \sqrt{x^2 + y^2} = x + y$$

$$x^2(x^2 + y^2) = (x + y)^2$$

The following symmetry tests do NOT indicate that the graph is symmetric:
 $(-r, \theta)$, $(-r, \pi - \theta)$ and $(r, \pi + \theta)$

- [a] Using the results above, along with the tests and shortcuts shown in lecture, determine if the graph is symmetric over the polar axis, $\theta = \frac{\pi}{2}$ and/or the pole. Summarize your conclusions in the table on the right.

NOTE: Run as FEW tests as needed to prove your conclusions are correct.

POLAR AXIS: $(r, -\theta)$ $r = 2 + 2\sin 3(-\theta)$

$r = 2 - 2\sin 3\theta$

$\theta = \frac{\pi}{2}$: $(r, \pi - \theta)$ $r = 2 + 2\sin 3(\pi - \theta)$

$r = 2 + 2\sin (3\pi - 3\theta)$

$r = 2 + 2[\sin 3\pi \cos 3\theta - \cos 3\pi \sin 3\theta]$

$r = 2 + 2\sin 3\theta$

Type of symmetry	Conclusion
Over the polar axis	NO CONCL
Over $\theta = \frac{\pi}{2}$	SYM
Over the pole	NO CONCL

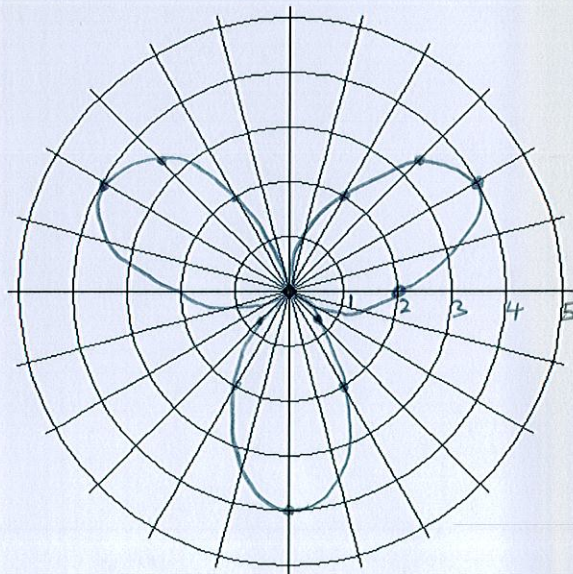
- [b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph)?

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

- [c] The table below shows the value of r for all values of θ in the first quadrant. Find additional values of r if necessary and sketch the graph of the equation.

Label the scale on your axes clearly.

θ	r	θ	r
0	2		
$\frac{\pi}{6}$	4	$-\frac{\pi}{6}$	0
$\frac{\pi}{4}$	≈ 3.4	$-\frac{\pi}{4}$	0.6
$\frac{\pi}{3}$	2	$-\frac{\pi}{3}$	2
$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$	4



Consider the conic with rectangular equation $7x^2 - 28x + 3y^2 + 30y + 40 = 0$.

SCORE: ____ / 20 PTS

- [a] What is the shape of the graph of the equation ?

ELLIPSE

- [b] Find the co-ordinates of the focus/foci.

$$7(x^2 - 4x + 4) + 3(y^2 + 10y + 25) = -40 + 28 + 75$$

$$7(x - 2)^2 + 3(y + 5)^2 = 63$$

$$\frac{(x-2)^2}{9} + \frac{(y+5)^2}{21} = 1$$

$$c^2 = 21 - 9 = 12$$

$$c = 2\sqrt{3}$$



FOCI
 $(2, -5 \pm 2\sqrt{3})$

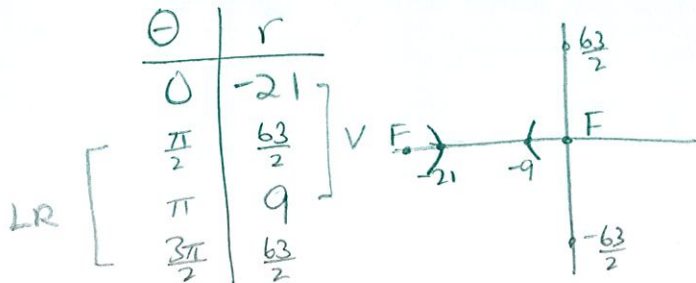
Consider the polar equation $r = \frac{63}{2-5\cos\theta} = \frac{\frac{63}{2}}{1-\frac{5}{2}\cos\theta}$

SCORE: ____ / 20 PTS

- [a] What is the shape of the graph of the equation?

$$e = \frac{5}{2} > 1 \rightarrow \text{HYPERBOLA}$$

- [b] Find the rectangular coordinates of the endpoints of all latera recta. Do NOT convert the equation to rectangular.



OTHER FOCUS $(-21-9, 0) = (-30, 0)$

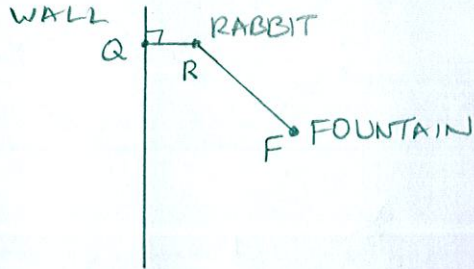
ENDPOINTS OF L.R.

$$(0, \pm \frac{63}{2}) \quad (-30, \pm \frac{63}{2})$$

A drinking fountain is 6 feet from the wall of a school building. A rabbit is running on the school grounds, so that it is always half as far from the wall as it is from the fountain. What is the shape of the rabbit's path?

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Draw a diagram and write algebraic equations involving distances to justify your answer.



$$RQ = \frac{1}{2} RF$$

$$2 = \frac{RF}{RQ} = e$$

$$2 > 1 \rightarrow \text{HYPERBOLA}$$